Atmospheric Aerosol Physics, Physical Measurements, and Sampling

Mechanical Properties

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Leibniz Association

ECAC
European Center for Aerosol Calibration

WCCAP
World Calibration Centre
for Aerosol Physics

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External Forces

External forces on a particles will result in a macroscopically directed particle motion.

Examples for external forces:

- Gravitational force

$$\boxed{\vec{F}_\mathrm{g} = \frac{\pi}{6} D_\mathrm{P}^3 \cdot \rho_\mathrm{P} \cdot \vec{g} \cdot (1 - \frac{\rho_\mathrm{G}}{\rho_\mathrm{P}})}$$

- Electrical force

$$\boxed{\vec{F}_\mathrm{e} = n_\mathrm{e} \cdot \mathrm{e} \cdot \vec{E}}$$

- Thermophoresis

$$\boxed{\vec{F}_\mathrm{th} = -\frac{3 \cdot \pi \cdot \eta^2 \cdot D_\mathrm{P} \cdot K_\mathrm{th}}{\rho_\mathrm{G}} \cdot \frac{\nabla T}{T}}$$

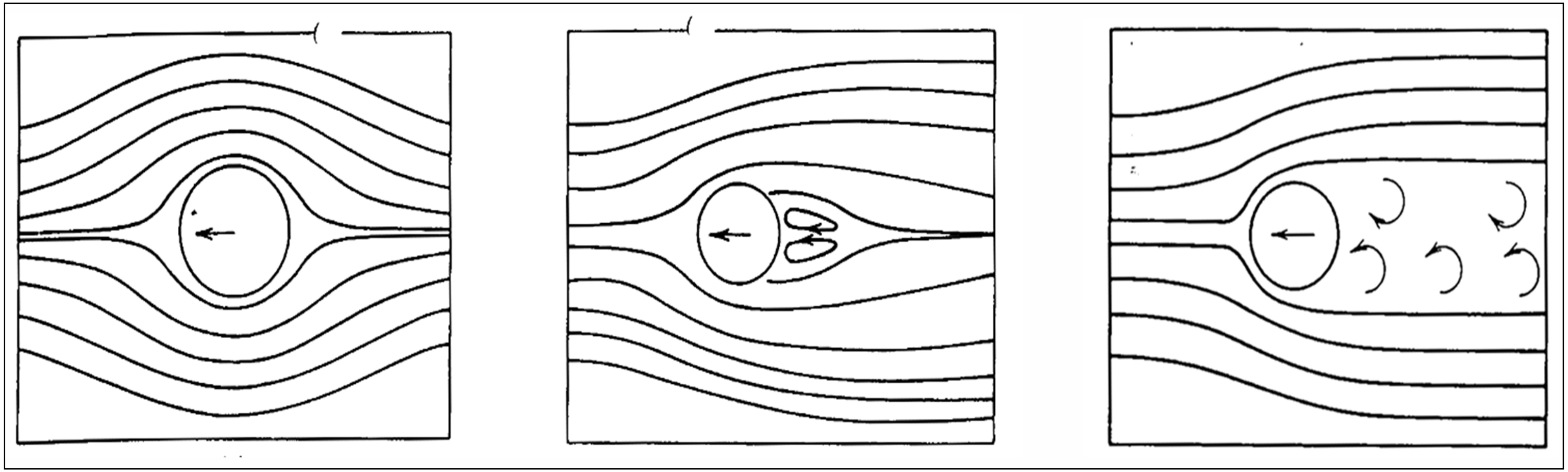
Particle Reynolds Number

The **Particle-Reynolds-Number** is an important quantity for the characterization of the mechanical properties of aerosol particles.

$$\boxed{\mathrm{Re_P} = \frac{\rho_\mathrm{G} u_\mathrm{P} D_\mathrm{P}}{\eta}}$$

- It characterizes the flow around an aerosol particle.
- Equal Particle-Reynolds-Numbers imply the same pattern of stream lines in the vicinity of particles with different size and in different gases.
- It is the most important property for the determination of the drag force a gas exerts on a suspended particle.

Flow around a sphere



Laminar flow, Re = 0.1

Turbulent flow, Re ≈ 2

Turbulent flow, Re ≈ 250

Hinds: Aerosol Technology: Properties, Behavior, and Measurement of Airborne Particles

Stokes' Law

Most particle movement takes place at low Particle-Reynolds-Numbers, as both, the flow velocity and the particle diameter are usually small.

Stokes' Law is a special solution of the momentum, i.e. the Navier-Stokes-equation.

Therefore, the following assumptions are made:

- incompressible flow
- steady state
- gas velocity equal to zero at the particle surface (no slip boundary)

The drag coefficient can be determined applying Newton's Law:

$$\boxed{C_{\mathrm{D}} = \frac{24}{\mathrm{Re}}}$$

The **drag force** results in:

$$\boxed{\vec{F}_{\mathrm{D}} = 3\pi \cdot \eta \cdot \vec{u}_P \cdot D_P}$$

The **temperature dependency** of the viscosity can be described as follows:

$$\boxed{\eta = \eta_0 \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{T_0 + 110.4\mathrm{K}}{T + 110.4\mathrm{K}}\right)}$$

Cunningham Correction Factor

For particle diameters D_P < 10000 nm, the gas velocity at the particle surface is not equal to zero, which results in a reduced drag force.

This effect is accounted for by the **Cunningham correction factor**

$$\boxed{C_\mathrm{C} = 1 + \frac{\lambda}{D_\mathrm{P}}\left(2.514 + 0.8 \cdot \exp\left(-0.55\frac{D_\mathrm{P}}{\lambda}\right)\right)}$$

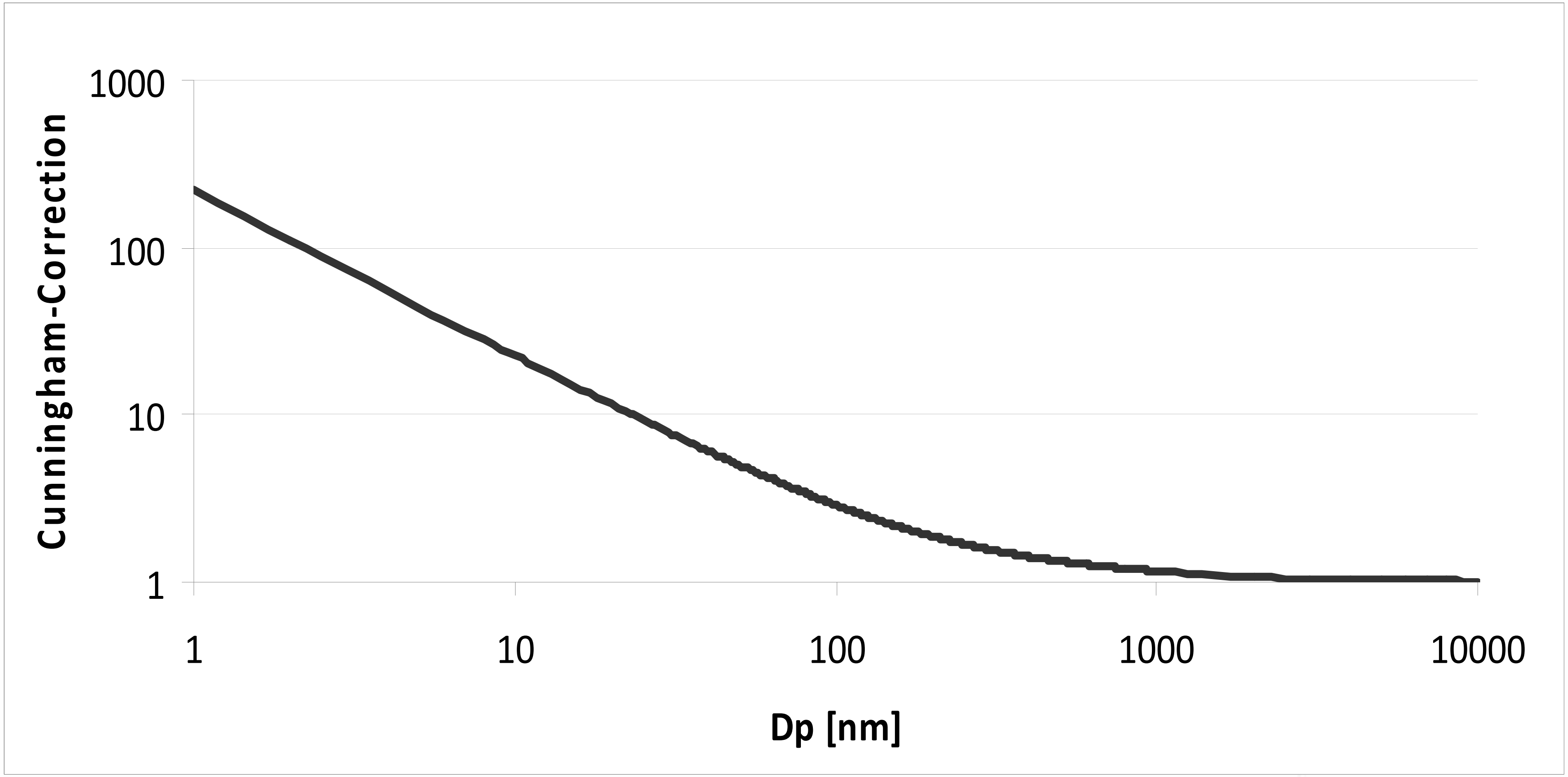
The **drag force** then becomes:

$$\boxed{\vec{F}_\mathrm{D} = \frac{3\pi \cdot \eta \cdot \vec{u}_\mathrm{P} \cdot D_\mathrm{P}}{C_\mathrm{C}}}$$

The Cunningham correction factor was determined empirically via the determination of the settling velocity of particles with known size and density.

For the pressure & temperature dependency of the Cunningham correction factor, the mean free path has to be corrected according to:

$$\boxed{\lambda = \lambda_0 \left(\frac{T}{T_0}\right)^2 \left(\frac{p_0}{p}\right)\left(\frac{T_0 + 110.4\mathrm{K}}{T + 110.4\mathrm{K}}\right)}$$



Cunningham correction factor as function of particle size

Mechanical Mobility

The **drag force** on a particle in an uniform motion results in:

$$\boxed{\vec{F}_{\mathrm{D}} = 3\pi \cdot \eta \cdot \vec{u}_P \cdot D_P}$$

In Stokes' Law the drag force is directly proportional to the relative velocity between the particle and the gas. This fact can be used to introduce the **mechanical mobility** *B* :

$$\boxed{B = \frac{\vec{u}_{\mathrm{P}}}{\vec{F}_{\mathrm{D}}} = \frac{C_{\mathrm{C}}}{3\pi \cdot \eta \cdot D_{\mathrm{P}}}}$$

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$$\boxed{C_{\mathrm{C}} = 1 + \frac{\lambda}{D_{\mathrm{P}}}(2.514 + 0.8 \cdot \exp(-0.55\frac{D_{\mathrm{P}}}{\lambda}))}$$

Sedimentation Velocity

An important application of Stokes' law is the determination of the **sedimentation velocity** u_s of aerosol particles. In this case, the drag forces F_D is equal to the gravitational force F_G acting on the particle.

$$\boxed{\begin{aligned}&\vec{F}_\mathrm{D} = \vec{F}_\mathrm{G}\\ &\frac{3\pi\cdot\eta\cdot\vec{u}_\mathrm{s}\cdot D_\mathrm{P}}{C_\mathrm{C}} = \frac{\pi\cdot\rho_\mathrm{P}\cdot D_\mathrm{P}^3\cdot\vec{g}}{6}(1-\frac{\rho_\mathrm{G}}{\rho_\mathrm{P}})\end{aligned}}$$

If the gas density is small compared to the particle density, the sedimentation velocity u_s can be calculated as follows:

$$\boxed{\vec{u}_\mathrm{s} = \frac{\rho_\mathrm{P} D_\mathrm{P}^2 C_\mathrm{C}\vec{g}}{18\eta}}$$

Settling velocity	
density = 1	
Dp (nm)	Us (m/s)
1	6,67E-09
10	6,82E-08
100	8,73E-07
1000	3,49E-05
10000	3,03E-03
100000	2,99E-01

Brownian Motion of Aerosol Particles

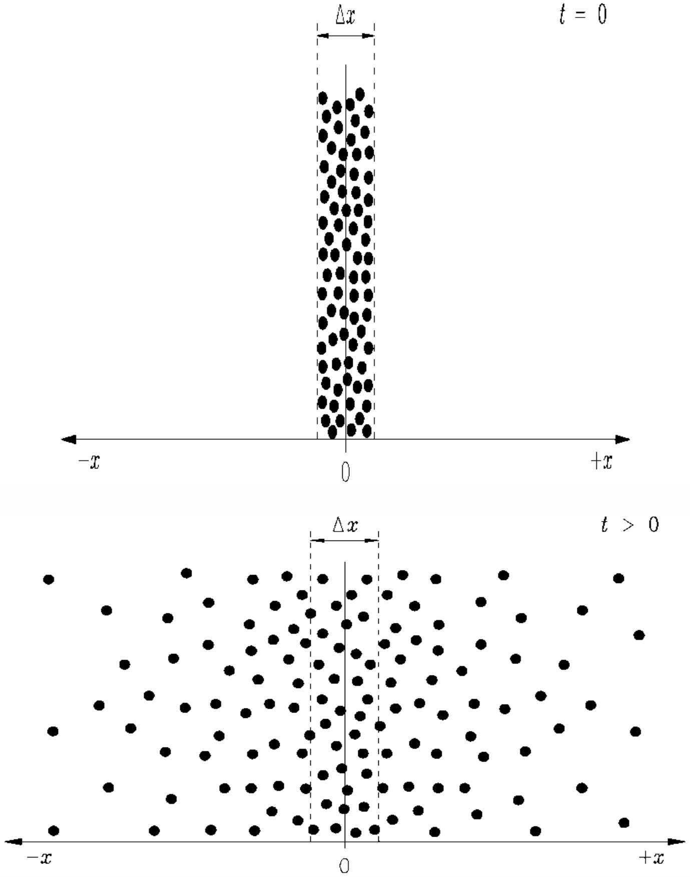

Figure: Hinds: Aerosol Technology: Properties, Behavior, and Measurement of Airborne Particles

Particle Diffusion

- The particle diffusion results from interactions between particles and gas molecules.
- Due to the impulse exchange molecules and particles, the Brownian motion of the molecules is transferred to the particles.
- The resulting particle motion is then called **Brownian particle motion**.
- A measure of the Brownian particle motion is the particle diffusion coefficient *D*.
- The Brownian particle motion is macroscopically non-directional.

$$\boxed{D = k \cdot T \cdot B}$$

Stokes Number

The Stokes number characterizes the particle inertia in a flow

$$\boxed{\mathrm{Stk} = \frac{\tau \cdot u_0}{D_{\mathrm{pipe}}}}$$

with

$$\boxed{\tau = \frac{\rho_{\mathrm{P}} \cdot D_{\mathrm{P}}^2 \cdot C_{\mathrm{C}}}{18\eta}}$$

τ … relaxation time

u_0 … wind velocity

D_{pipe} … tube diameter

The Stokes Number is the ratio between the particle stopping distance to characteristic dimensions of the flow profile.

Flow-Reynolds-Number

The **Flow-Reynolds-Number** depends mainly on the flow rate and the tube diameter

$$\boxed{\mathrm{Re}_{\mathrm{flow}} = \frac{\rho_{\mathrm{gas}} \cdot \overline{u}_{\mathrm{flow}} \cdot D_{\mathrm{pipe}}}{\eta}}$$

ρ_{gas} … gas density
u_{flow} … flow velocity
D_{pipe} … tube diameter
η … dynamic viscosity

Example

Particle diameter nm	Relaxation time s	Stopping distance m	Stokes number
10	6,95E-09	9,23E-09	2,31E-06
100	8,90E-08	1,18E-07	2,95E-05
1000	3,56E-06	4,72E-06	1,18E-03
10000	3,09E-04	4,11E-04	1,03E-01
100000	3,05E-02	4,04E-02	1,01E+01
Density: ρ_P	2000 kg/m^3		
Tube diameter: D_t	0.004 m		
Tube velocity: u_t	1.33 m/s		
	(5 l/min in ¼” tube)		

Particle Losses

Particle losses in transport systems can occur due to:

- Sedimentation in horizontal or sloping pipes (coarse particles)
- Inertia in bends (coarse particles)
- Diffusion (ultrafine particles)

Particle Losses in Pipes

For coarse particles > 1 µm

- Pipes should be vertically orientated.
- In cases when horizontal or sloping pipes cannot be avoided, the flow should be high.
- Bends should be avoided.
- Highly turbulent flows cause increased inertial losses.

For ultrafine particles < 100 nm

- Pipes should be kept as short as possible.
- The transport system should be designed for a laminar flow with the optimum Reynolds number of 2000.
- Turbulent flows should be avoided, because of higher diffusional particle losses.